Student name: _____   Number:_____

_____

1) Determine which of the following functions is Riemann integrable. Justify

   a) $f(x) = \sin x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

   b) $f(x) = \tan x \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

   c) $f(x) = \begin{cases} 2 & -2 \leq x < 0 \\ 0 & 0 \leq x \leq 1 \\ 5 & 1 < x \leq 2 \end{cases}$

   d) $f(x) = \begin{cases} 2 - x & -2 \leq x < 0 \\ x^2 & 0 \leq x \leq 3 \end{cases}$

2) If $f \in \mathcal{R}[a, b]$, prove that $f^2 \in \mathcal{R}[a, b]$.

3) Suppose that f is continuous on $[a, b]$, $f(x) \geq 0 \ \forall \ x \in [a, b]$ and $\int_a^b f = 0$.
   prove that $f(x) = 0 \ \forall \ x \in [a, b]$.

4) Solve just one of the following questions

   a) If $f: [a, b] \to \mathbb{R}$ is continuous on $[a, b]$, then $f \in \mathcal{R}[a, b]$.

   b) If $f: [a, b] \to \mathbb{R}$ is monotone on $[a, b]$, then $f \in \mathcal{R}[a, b]$.