

Philadelphia University
Faculty of Science
Department of Basic Sciences and Mathematics
Real Analysis 2 Second Exam

Student name: _____

Number: _____

1) Determine which of the following functions is Riemann integrable. Justify

a) $f(x) = \sin x, x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

b) $f(x) = \tan x, x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

c) $f(x) = \begin{cases} 2 & -2 \leq x < 0 \\ 0 & 0 \leq x \leq 1 \\ 5 & 1 < x \leq 2 \end{cases}$

d) $f(x) = \begin{cases} 2-x & -2 \leq x < 0 \\ x^2 & 0 \leq x \leq 3 \end{cases}$

2) If $f \in \mathcal{R}[a, b]$, prove that $f^2 \in \mathcal{R}[a, b]$.

3) Suppose that f is continuous on $[a, b]$, $f(x) \geq 0 \forall x \in [a, b]$ and $\int_a^b f = 0$.
prove that $f(x) = 0 \forall x \in [a, b]$.

4) Solve just one of the following questions

a) If $f: [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then $f \in \mathcal{R}[a, b]$.

b) If $f: [a, b] \rightarrow \mathbb{R}$ is monotone on $[a, b]$, then $f \in \mathcal{R}[a, b]$.